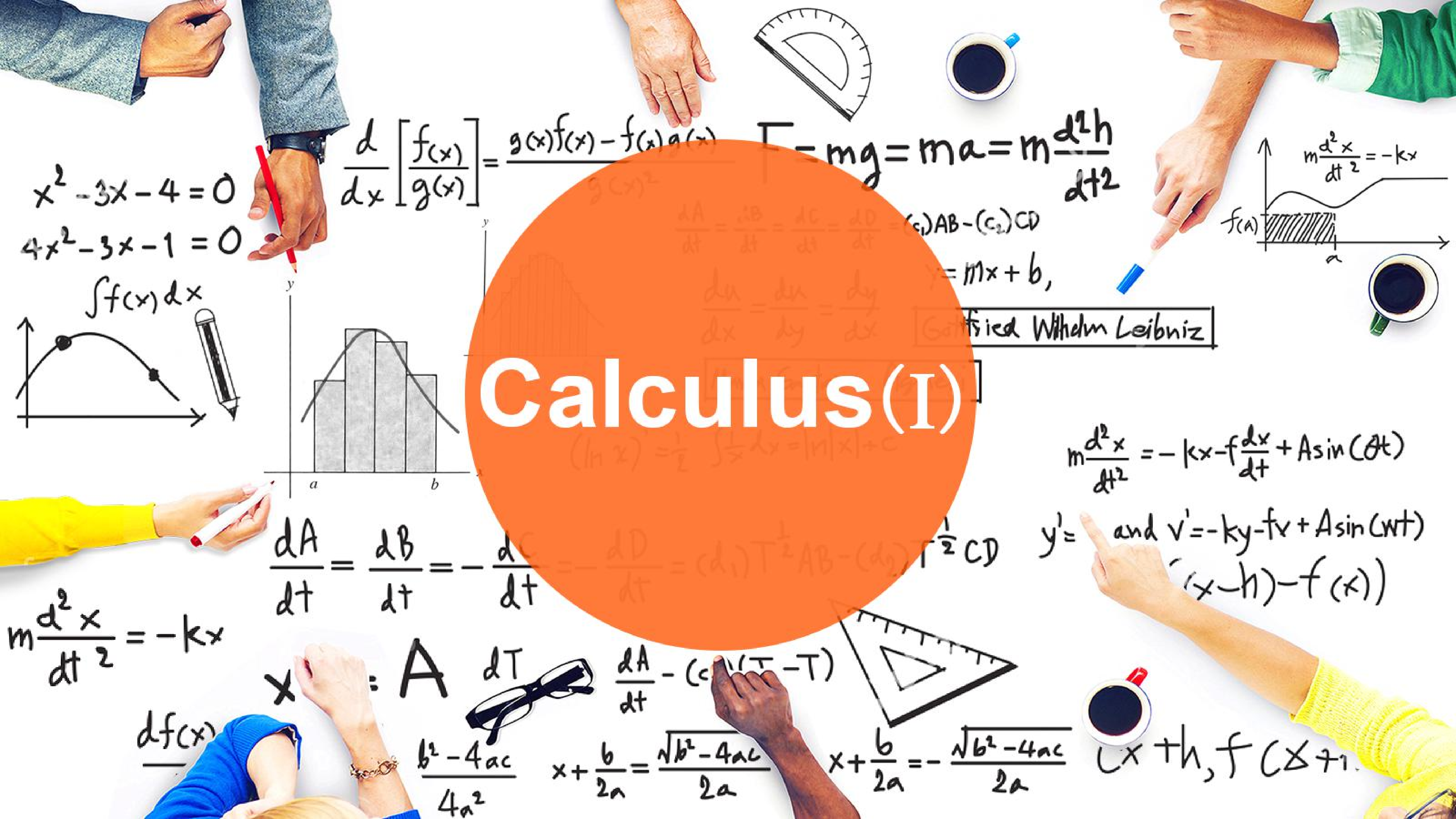

x² -3x - 4 = 0
4x² - 3x - 1 = 0
∫f(x) dx
y = mx + b,
Gottfried Wilhelm Leibniz
m d²x/dt² = -kx
m d²x/dt² = -kx - f dx/dt + A sin(θt)
and v' = -ky - fv + A sin(wt)
x + b/2a = √(b² - 4ac)/2a
x + b/2a = -√(b² - 4ac)/2a

Calculus(I)

3.4 Practical Problems

Lecturer: Xue Deng

In practical life, there are many practical optimization problems, how to deal with the practical problems?

Suggest: step-by step method.

Solving Method: Computation steps

Step1 Draw a picture and assign appropriate variables.

Step2 Write a formula for the objective function Q.

Step3 Express Q as a function of a single variable.

Step4 Find the critical points (end, stationary and singular points).

Step5 Determine the maximum or minimum.

If there is a unique stationary point about the objective function, the function value of this point is the desirable maximum (minimum).

Example 1

Find the greatest volume that a right circular cylinder can have if it is inscribed in a sphere of radius R.

Let the height of cylinder be $2h$, radius be r , volume be V

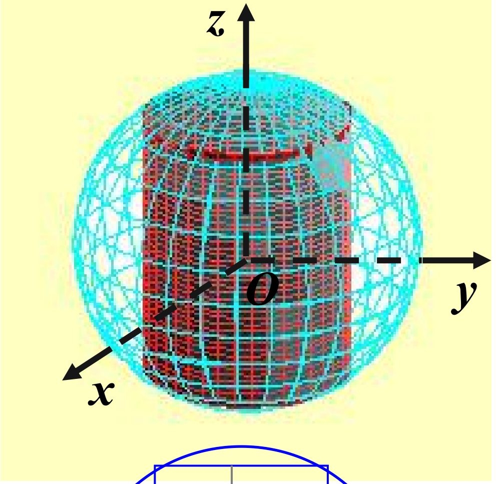
First: Objective function $V=\pi r^2 \cdot 2h$

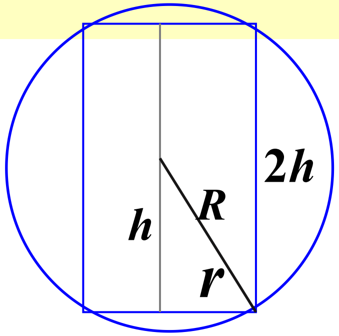
By $r^2+h^2=R^2$,

obtain $V=2\pi(R^2-h^2)\cdot h$, $0<h<R$

Then: Find maximum point

$$V_h'=2\pi(R^2-3h^2)$$





Example 1

Find the greatest volume that a right circular cylinder can have if it is inscribed in a sphere of radius *r*.

Let $V_h' = 0$, obtain $h = \frac{R}{\sqrt{3}}$ (delete negative value) $V_h' = 2\pi(R^2 - 3h^2)$

(Unique stationary point:the maximum volume of the cylinder must be obtained.)

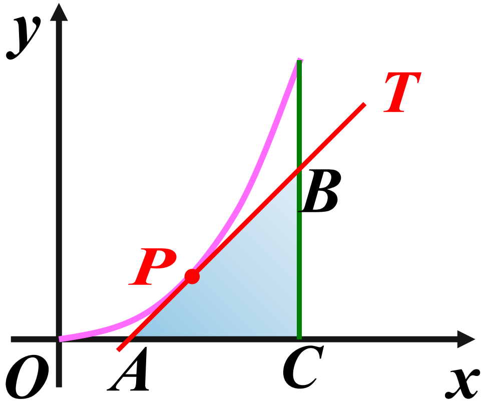
So the unique stationary point $h = \frac{R}{\sqrt{3}}$ is the maximum value point.

The maximum volume: $V = 2\pi(R^2 - \frac{R^2}{3}) \cdot \frac{R}{\sqrt{3}} = \frac{4\pi}{3\sqrt{3}} R^3.$

Example 2

The curved triangle area bounded by $\begin{cases} y=0 \\ x=8 \\ y=x^2 \end{cases}$. Find a point $P \in y=x^2$ such that triangle area bounded by the tangent line through P, y=0 and x=8 is the maximum.

As shown, let the tangent point $P(x_0, y_0)$,



the tangent line PT:

$$y - y_0 = 2x_0(x - x_0),$$

$\because\ y_0 = x_0^2, \quad \therefore A(\frac{1}{2}x_0,\ 0), \quad C(8,\ 0), \quad B(8,\ 16x_0 - x_0^2)$

Example 2

$$\therefore S_{\Delta ABC}=\frac{1}{2}(8-\frac{1}{2}x_0)(16x_0-x_0^2)=x_0(8-\frac{x_0}{2})^2 \quad (0<x_0<8)$$

$$S'_{x_0}=\left(8-\frac{x_0}{2}\right)^2-x_0\left(8-\frac{x_0}{2}\right)=\left(8-\frac{x_0}{2}\right)\left(8-\frac{3x_0}{2}\right)$$

Let $S'_{x_0}=0$, obtain $x_0=\frac{16}{3}$, $x_0=16$ (delete).

Unique stationary point

The maximum area is obtained, sopoint $P\left(\frac{16}{3},\left(\frac{16}{3}\right)^2\right)$ is the final result.

So $S(\frac{16}{3})=\frac{4096}{27}$ is the maximum area.

Summary of Practical Problems

The steps to solve the practical optimization problems:

- **Step1** Draw a picture and assign appropriate variables.
- **Step2** Write a formula for the objective function Q.
- **Step3** Express Q as a function of a single variable.
- **Step4** Find the critical points (end, stationary and singular points).
- **Step5** Determine the maximum or minimum.

Questions and Answers

A rectangular box is to be made from a piece of cardboard 24 inches long and 9 inches wide by cutting out identical squares from the four corners and turning up the sides.

Q1: Find the dimensions of the box of maximum volume.

Q2: What is this volume? (See book P167)

Questions and Answers

Let x be the width of the square to be cut out and V the volume of the resulting box .

Then $V = x(9-2x)(24-2x) = 216x - 66x^2 + 4x^3 \ (0 \le x \le 4.5)$

$V_x' = 216 - 132x + 12x^2 = 12(x-9)(x-2)$

Let $V_x' = 0$, obtain $x = 2, x = 9$(delete)

So there are only there critical point: 0 , 2 and 4.5 .

$V(0) = 0$, $V(4.5) = 0$, $V(2) = 200$

Then $x = 2$ is the final result. The volume is 200.

The box is 20 inches long, 5 inches wide, and 2 inches deep.

Practical Problems